

# Quantum computation in a quantum-dot-Majorana-fermion hybrid system

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We propose a scheme to implement universal quantum computation in a quantum-dot-Majorana-fermion hybrid system. Quantum information is encoded on pairs of Majorana fermions, which live on the interface between topologically trivial and nontrivial sections of a quantum nanowire deposited on an  $s$ -wave superconductor. Universal single-qubit gates on topological qubit can be achieved. A measurement-based two-qubit Controlled-Not gate is produced with the help of parity measurements assisted by the quantum-dot and followed by prescribed single-qubit gates. The parity measurement, on the quantum-dot and a topological qubit, is achieved by the Aharonov-Casher effect.

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## I. INTRODUCTION

Recently, physical implementation of quantum computers has attracted much attention. One of the main difficulties of scalable quantum computation is decoherence of quantum information. A promising strategy against decoherence is based on the topological idea [1] where gate operations depend only on global features of the control process, and thus largely insensitive to local noises. Topological ordered states emerge as a new kind of states of quantum matter beyond the description of conventional Landau theory [2]. A paradigmatic system for the existence of anyons is a kind of so-called fractional quantum Hall states [2]. Alternatively, artificial spin lattice models are also promising for observing these exotic excitations, e.g., Kitaev models [3] are most famous for demonstrating anyonic statistics [4–6] and braiding operations for topological quantum computation.

For universal quantum computation, one needs non-Abelian anyon to serve as qubit. With the potential applications in topological quantum computation, Majorana fermions (MF) with non-Abelian statistics have attracted strong renewed interests. MF are a kind of self-conjugate quasiparticles induced from a vortex excitation in  $p_x + ip_y$  superconductor [1]. However, due to the instability of the  $p$ -wave superconducting states, its implementation remains an experimental challenge. Therefore, setups supporting MF with  $s$ -wave superconductor [7–13], which is more stable, by proximity effect is more preferable. More recently, it is recognized that topologically protected states may be most easily engineered in 1D semiconducting nanowires deposited on an  $s$ -wave superconductor [14–17], which provides the first realistic experimental setting for Kitaev's 1D topological superconducting state [18].

Although not universal for quantum computation, parity protected quantum logical operations schemes with a pair of MF as a topological qubit are proposed [19–25], which mainly focus on braiding operations of single qubit [19–22] and quantum information transfer between a topological qubit and a quantum bus [23–25]. As the absence of topologically ordered system that is universal for quantum computation and extremely difficult to perform certain topology changing opera-

tions, present research of universal quantum computation with MF, as well as other exotic topological qubits, need to supplement braiding operations with topologically unprotected operations. These operations can be error-corrected for a high error-rate threshold of approximately 0.14 [26]. However, such a high error threshold may still prove difficult using unprotected operations within a topological system. Therefore, a topological quantum bus, usually in a topological and conventional qubits hybrid architecture, would be of great help, where error rates below 0.14 have already been achieved [27].

Here, we propose a scheme to implement universal quantum computation in a quantum-dot-MF hybrid system. The architecture consists of 1D semiconducting wires deposited on an  $s$ -wave superconductor, under certain conditions, the end-points of such wires support localized zero-energy MF. Qubit is encoded on a pairs of MF (topological qubit), universal single-qubit gates in encoded qubit can be achieved. In particular, the realization of parity measurements on a quantum-dot (QD) and a topological qubit, together with control and measurement on the QD state and single-qubit gates on the target topological qubit, is able to generate a two-qubit Controlled-Not (CNOT) gate [28]. In this sense, this scheme is the measurement-based quantum computation.

## II. SETUP

The setup we consider is shown in Fig. 1, which is a spin-orbit coupled semiconducting wire deposited on an  $s$ -wave superconductor, which, together with Josephson junctions, form a superconducting flux qubit configuration. Applying a magnetic field perpendicular to the superconductor surface, the Hamiltonian describing such a wire is [14]

$$H = \int \left[ \psi_x^\dagger \left( -\frac{\hbar^2 \partial_x^2}{2m} - \mu - i\hbar u \hat{e} \cdot \sigma \partial_x + V_B \sigma_z \right) \psi_x + (|\Delta| e^{i\varphi} \psi_{\downarrow x} \psi_{\uparrow x} + h.c.) \right] dx, \quad (1)$$

where  $\psi_{\alpha x}$  corresponds to electrons with spin  $\alpha$ , effective mass  $m$ , and chemical potential  $\mu$ ; the third term denotes spin-orbit coupling with  $u$  the strength, and  $\sigma = (\sigma_x, \sigma_y, \sigma_z)$  is the

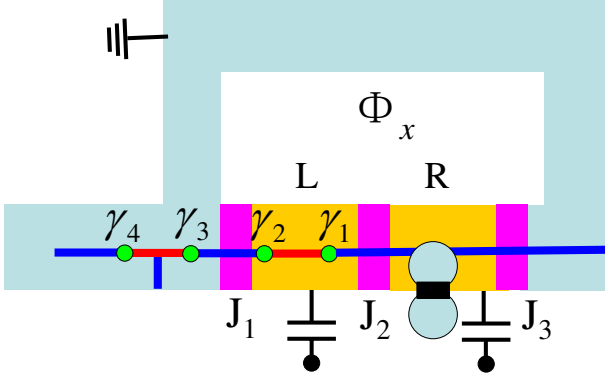


FIG. 1: (Color online) Superconducting flux qubit with three Josephson junctions (pink) and an enclosing magnetic flux  $\Phi_x$ . MF (green dots) are induced at the interface between a topologically trivial (blue) and a topologically nontrivial (red) section of a quantum nanowire. The right superconducting island is located by a semiconductor double quantum-dot qubit. Gate electrodes (not shown) can be used to move the MF along the wire.

vector of Pauli matrices; the fourth term represents the energy shift due to the magnetic field; and the terms in the second line are the spin-singlet pairing from the  $s$ -wave superconductor via proximity effect. The interplay of Zeeman effect, spin-orbit coupling, and the proximity to an  $s$ -wave superconductor drive the wire into a chiral  $p$ -wave superconducting state [14, 15]. For  $|\mu| < \mu_c = \sqrt{V_B^2 - |\Delta|^2}$  the topological phase with end Majoranas emerges, or a topologically trivial phase. Thus, applying a gate voltage uniformly allows one to create or remove the MF. To avoid gap closure, a “keyboard” of local tunable gate electrodes to the wire [16] is used to control whether a region of the wire is topological or not. For InAs quantum nanowire, assuming  $|V_B| \sim 2|\Delta|$  and  $\hbar u \sim 0.1\text{eV}\text{\AA}$ , the gap for a  $0.1\mu\text{m}$  wide gate is of order 1K [16]. The QD used here is semiconductor double quantum dot molecule with one dot deposit on the superconductor of the flux qubit circuit while the other is not. We assume here that there is a galvanic isolation between the superconductor and semiconductor, so that there is no charge transfer between them. Remarkably, one can also realize this QD using InAs nanowires [29] for supporting MF, and thus reduce the experimental difficulties.

### III. MF AS QUBIT

A pair of MF can be combined into a complex fermion. The fermion parity operator  $n_p$  has eigenvalues  $-1$  and  $+1$  in states  $|0\rangle$  and  $|1\rangle$ , respectively. The two states  $|0\rangle$  and  $|1\rangle$  of a pair of MF differ by fermion parity, which prevents the creation of a coherent superposition. For the purpose of quantum computation, we combine four Majorana bound states to form a topological qubit so that coherent superpositions are allowed. Without loss of generality we can assume that the joint fermion parity is even, i.e., the two states of the topological qubit are encoded as  $|00\rangle$  and  $|11\rangle$ .

#### A. Qubit readout

We now turn to the problem of reading out the two qubit states. As the states of the two pairs of MF in a topological qubit are always the same, detecting one of them can then fulfill the purpose of distinguishing them. The two states  $|0\rangle$  and  $|1\rangle$  are different in fermion parity  $n_p$ , so they can be distinguished by  $n_p$ . Without loss of generality, we choose to detect the pair of  $\gamma_1$  and  $\gamma_2$  while move  $\gamma_3$  and  $\gamma_4$  out of the flux qubit circuit, as shown in Fig. 1. Note that the wire is not interrupted by the junctions providing that the junctions’ thickness is much smaller than  $\xi$ . To measure the parity of  $n_p$ , we make use of the suppression of macroscopic quantum tunneling by the Aharonov-Casher effect [30]: a vortex encircling a superconducting island picks up a phase increment  $\phi = \pi q/e$  determined by the total charge  $q$  coupled capacitively to the superconductor, which includes both the charge on the superconducting island itself and on a nearby gate electrode.

Following Ref. [30], we consider a superconducting flux qubit with three Josephson junctions, as shown in Fig. 1. The phase differences across the three junctions 1, 2, 3 sum to  $\phi_1 + \phi_2 + \phi_3 = 2\pi\Phi_x/\Phi_0$  with  $\Phi_x$  and  $\Phi_0 = h/2e$  being the external magnetic flux and flux quantum. Junctions 1 and 3 are assumed to have the same critical current  $I_c$  while junction 2 has critical current  $\alpha I_c$ . The charging energy  $E_C = e^2/(2C)$  of the islands with capacitance  $C$  is much larger than the Josephson coupling energy  $E_J = \Phi_0 I_c/(2\pi)$ , to ensure that the qubit works in the flux regime. Assuming that the size of the qubit is sufficiently small, thus the flux generated by the supercurrent can be neglected. Therefore, the enclosed flux of the qubit contour equals the external flux. Then, the superconducting energy of the flux qubit reads

$$U = -E_J \left[ \cos \phi_1 + \cos \phi_3 + \alpha \cos \left( 2\pi \frac{\Phi_x}{\Phi_0} - \phi_1 - \phi_3 \right) \right], \quad (2)$$

which exists two degenerate states  $|L\rangle$  and  $|R\rangle$  distinguished by the phases of the order parameter on the left and right islands; the supercurrent flows to the left or to the right in state  $|L\rangle$  and  $|R\rangle$ . The states  $|L\rangle$  and  $|R\rangle$  correspond the potential energy minima in Eq. (2), the tunneling between the two states requires quantum phase slips.

As all phases differ by  $2\pi$  are equivalent, for  $\alpha > 1$  the energy minima are connected by two tunneling paths, differ by an increment of  $2\pi$  in  $\phi_1$  and  $-2\pi$  in  $\phi_3$ . The difference between the two tunneling paths amounts to the circulation of a Josephson vortex around both islands  $L$  and  $R$ . Their phase difference is  $\psi = \pi q/e$ , with  $q = \sum_{i=L,R} (en_p^i + q_i)$  being the total charge on the two islands  $en_p^i$  and gate capacitors  $q_i$ . The two paths have the same amplitude, because of the left-right symmetry of the flux qubit ( $I_{c1} = I_{c3} = I_c$ ). The interference produces an oscillatory tunnel splitting of the two levels of the flux qubit

$$\Delta = \Delta_0 \left| \cos \left( \frac{\psi}{2} \right) \right|, \quad (3)$$

where  $\Delta_0$  is the tunnel splitting associated with one path.

Therefore, if  $q$  is an odd (even) multiple of the electron charge  $e$ , the two tunneling paths interfere destructively (constructively), so the tunnel splitting is minimum (maximal).

As we only need to distinguish maximal from minimal tunnel splitting, the flux qubit does not need to have a large quality factor. In addition,  $\Delta_0 \simeq 100\mu\text{eV} \simeq 1\text{K}$  for parameters in typical experiments of flux qubits [30], which should be readily observable by microwave absorption. To make sure the total charge is solely comes from the two superconducting islands, one would first calibrate the charge on the gate capacitor to zero, i.e.,  $q_i = 0$ , by maximizing the tunnel splitting in the absence of vortices in the island. The read-out is nondestructive, which is necessary for our proposed way of implementation the two-qubit CNOT gate. Meanwhile, it is insensitive to sub-gap excitations in the superconductor as they do not change the fermion parity.

### B. Single-qubit gates

Pauli operators in the topological qubit basis can be expressed in terms of the  $\gamma$  operators as follows  $\sigma_x = -2i\gamma_2\gamma_3$ , and  $\sigma_z = -2i\gamma_1\gamma_2$ . Therefore, braiding can generate the unitary operations  $\exp[\pm(i\pi/4)\sigma_k]$  ( $k = x, z$ ). Providing the ability of braiding of neighboring MF in the proposed setup [16], one can generate such gate in a topological protected manner. For a universal quantum computer, one needs additionally to be able to perform single qubit rotations of the form

$$|0\rangle|0\rangle + |1\rangle|1\rangle \mapsto e^{-i\pi/4}|0\rangle|0\rangle + e^{i\pi/4}|1\rangle|1\rangle, \quad (4)$$

which is not topologically protected. This gate can be performed by appropriately adjust the external magnetic field of the flux qubit [20].

### C. Joint MF-QD parity measurement

As shown in Fig 1,  $q$  can also have a quantum component: charge comes from the QD. Logical basis of QD can be defined as semiconductor charge qubit  $|0\rangle = |0\rangle \otimes |1\rangle$  and  $|1\rangle = |1\rangle \otimes |0\rangle$  with the electron occupies the lower and upper dot, respectively. Therefore, the qubit basis states correspond to the electron parity on the upper dot enclosed by the Josephson vortex circulation. To read-out the MF state, one can simply set the QD to  $|0\rangle$  so that its charge do not interrupt the measurement. For our purpose, we are also interest in the joint parity of QD and a pair of MF. Indeed, the flux qubit splitting energy  $\Delta$  is the same for combined topological-QD qubit states with equal joint parity [24]. Thus, measurement of the flux qubit splitting energy is equivalent to a joint parity measurement to the states of QD and a pair of MF.

### D. CNOT gate

We next proceed to implement a CNOT gate between two topological qubits with the help of QD as an auxilia. Here we

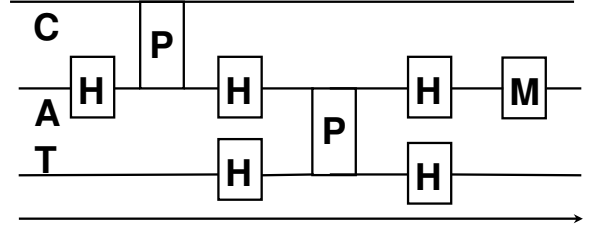


FIG. 2: Measurement-based CNOT gate for two topological qubits. Capital letters "A" represents ancilla of QD, "H" is the Hadamard gate, "C" and "T" represent the control and target qubit, respectively. The measurement "M" results of "A" together with the outcomes of the two joint parity measurements "P" determine which operation one has to apply on the "C" and "T" qubit in order to complete the CNOT gate. The arrowed line in the bottom represents the sequence of the process.

TABLE I: Correspondence between the measurement results and the gates operated on the control and target qubits. "0" and "1" represent odd and even parity, respectively.

"P <sub>1</sub> "	"P <sub>2</sub> "	result of "M"	gate on "C"	gate on "T"
1	1	$ 0\rangle_A$	I	I
1	1	$ 1\rangle_A$	I	X
1	0	$ 0\rangle_A$	Z	I
1	0	$ 1\rangle_A$	Z	X
0	1	$ 0\rangle_A$	I	X
0	1	$ 1\rangle_A$	I	I
0	0	$ 0\rangle_A$	Z	X
0	0	$ 1\rangle_A$	Z	I

propose a measurement-based CNOT gate operation [28]. The relevant operations are single-qubit rotations, single-QD rotations/measurements, and effective joint parity measurements for QD and a qubit. The circuit for the CNOT gate is depicted in Fig. 2. The auxiliary QD is initially prepared in the state of  $|0\rangle_A$ . After a Hadamard gate on the QD, the first joint parity measurement  $P_1$  in Fig. 2 is implemented on the QD and a pair of MF from "C" qubit. After Hadamard rotation of the QD and the target qubit, the second parity measurement  $P_2$  is implemented on the QD and a pair of MF in the "T" qubit. Then we rotate back the QD and the target qubit state by Hadamard gate. The last step is the measurement of the QD in the  $\{|0\rangle, |1\rangle\}$  basis. The two parity measurement results, together with the measurement result of the QD determine which single-qubit gates to be operated on the control and target qubits to generate a CNOT gate. The relationship between the measurement results and the gates to be operated is summarized in the table I. After completing the required gates on the corresponding qubits, it is straightforward to check that the process is a CNOT gate operation between the two qubits.

#### IV. DISCUSSION AND CONCLUSION

Before concluding, we want to emphasize that our scheme have the following distinct merits. (1) As in Ref. [24], one can host both MF and QD using a single InAs nanowires [29], which may thus reduce the technical challenges of implementation. (2) Here, QD serves as ancillary qubit. Therefore, only braiding between MF within a topological qubit is needed, since braiding of two MF from different topological qubits will be experimentally challenging. (3) Measurement of topological qubit uses the Aharonov-Casher effect [30], which is nondestructive. (4) Parity measurement on QD and a certain topological qubit can be achieved by moving the topological

qubit into the measurement circuit by local tunable gate on quantum wire [20].

In summary, we propose a scheme to implement universal quantum computation in a quantum-dot-MF hybrid system. Universal single-qubit gates on topological qubit can be achieved. A measurement-based two-qubit Controlled-Not gate is produced with the help of joint parity measurements and followed by prescribed single-qubit gates.

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